CX. *Of the Irregularities in the Motion of a Satellite arising from the spheroidical Figure of its Primary Planet: In a Letter to the Rev.* James Bradley *D. D. Astronomer Royal, F. R. S. and Member of the Royal Academy of Sciences at* Paris; *by Mr.* Charles Walmesley, *F. R. S. and Member of the Royal Academy of Sciences at* Berlin, *and of the Institute of* Bologna.

Reverend Sir,

Read Dec. 14, 1758.

SINCE the time that astronomers have been enabled by the perfection of their instruments to determine with great accuracy the motions of the celestial bodies, they have been solicitous to separate and distinguish the several inequalities discovered in these motions, and to know their cause, quantity, and the laws according to which they are generated. This seems to furnish a sufficient motive to mathematicians, wherever there appears a cause capable of producing an alteration in those motions, to examine by theory what the result may amount to, though it comes out never so small: for as one can seldom depend securely upon mere guess for the quantity of any effect, it must be a blameable neglect entirely to overlook it without being previously certain of its not being worth our notice.

Finding therefore it had not been considered what effect the figure of a planet differing from that of a

ſphere might produce in the motion of a ſatellite revolving about it, and as it is the caſe of the bodies of the Earth and Jupiter which have ſatellites about them, not to be ſpherical but ſpheroidical, I thought it worth while to enter upon the examination of ſuch a problem. When the primary planet is an exact globe, it is well known that the force by which the revolving ſatellite is retained in its orbit, tends to the center of the planet, and varies in the inverſe ratio of the ſquare of the diſtance from it; but when the primary planet is of a ſpheroidical figure, the ſame rule then no longer holds: the gravity of the ſatellite is no more directed to the center of the planet, nor does it vary in the proportion above-mentioned; and if the plane of the ſatellite's orbit be not the ſame with the plane of the planet's equator, the protuberant matter about the equator will by a conſtant effort of its attraction endeavour to make the two planes coincide. Hence the regularity of the ſatellite's motion is neceſſarily diſturbed, and though upon examination this effect is found to be but ſmall in the moon, the figure of the earth differing ſo little from that of a ſphere, yet in ſome caſes it may be thought worth notice; if not, it will be at leaſt a ſatisfaction to ſee that what is neglected can be of no conſequence. But however inconſiderable the change may be with regard to the moon, it becomes very ſenſible in the motions of the ſatellites of Jupiter both on account of their nearer diſtances to that planet when compared with its ſemidiameter, as alſo becauſe the figure of Jupiter ſo far recedes from that of a ſphere. This I have ſhewn and exemplified in the fourth ſatellite; in which caſe indeed the computation is more

exact

exact than it would be for the other ſatellites: for as my firſt deſign was to examine only how far the moon's motion could be affected by this cauſe, I ſuppoſed the ſatellite to revolve at a diſtance ſomewhat remote from the primary planet, and the difference of the equatoreal diameter and the axis of the planet not to be very conſiderable. There likewiſe ariſes this other advantage from the preſent theory, that it furniſhes means to ſettle more accurately the proportion of the different forces which diſturb the celeſtial motions, by aſſigning the particular ſhare of influence which is to be aſcribed to the figure of the central bodies round which thoſe motions are performed.

I have added at the end a propoſition concerning the diurnal motion of the earth. This motion has been generally eſteemed to be exactly uniform; but as there is a cauſe that muſt neceſſarily ſomewhat alter it, I was glad to examine what that alteration could amount to. If we firſt ſuppoſe the globe of the earth to be exactly ſpherical, revolving about its axis in a given time, and afterwards conceive that by the force of the ſun or moon raiſing the waters its figure be changed into that of a ſpheroid, then according as the axis of revolution becomes a different diameter of the ſpheroid, the velocity of the revolution muſt increaſe or diminiſh: for, ſince ſome parts of the terraqueous globe are removed from the axis of revolution and others depreſſed towards it, and that in a different proportion as the ſun or moon approaches to or recedes from the equator, when the whole quantity of motion which always remains the ſame is diſtributed through the ſpheroid, the velocity of the diurnal rotation cannot be conſtantly the ſame. This

 variation

variation however will ſcarce be obſervable, but as it is real, it may not be thought amiſs to determine what its preciſe quantity is.

I am ſenſible the following theory, as far as it relates to the motion of Jupiter's ſatellites, is imperfect and might be proſecuted further; but being hindered at preſent from ſuch purſuit by want of health and other occupations, I thought I might ſend it you in the condition it has lain by me for ſome time. You can beſt judge how far it may be of uſe, and what advantage might ariſe from further improvements in it. I am glad to have this opportunity of giving a freſh teſtimony of that regard which is due to your diſtinguiſhed merit, and of profeſſing myſelf with the higheſt eſteem,

Reverend Sir,

Your very humble Servant,

Bath, Oct. 21, 1758.

C. Walmeſley.

Lemma I.

Invenire gravitatem corporis longinqui ad circumferentiam circuli ex particulis materiæ in duplicatâ ratione diſtantiarum inverſè attrahentibus conſtantem.

Esto NIK (*Vid.* Tab. xxxiii. *Fig.* 1.) circumferentia circuli, in cujus puncta omnia gravitet corpus longinquum S locatum extra planum circuli. In hoc planum agatur linea perpendicularis SH, et per circuli centrum X ducatur recta HXK ſecans circulum in I et K, et SR parallela ad HX: producatur autem SH ad diſtantiam datam SD, et agantur rectæ DC,

DC, XC, ipsis HX, SD, parallelæ. Tum ductâ chordâ quavis MN ad diametrum IK normali eamque secante in L, ex punctis M, N, demittantur in SR perpendiculares MR, NR, concurrentes in R; junctisque SM, SN, erit $SM = SN$, $MR = NR$, $SR = HL$. Dicantur jam SD, k; HX sive DC, h; XL, x; CX, z; XI, r; eritque $HL = h - x$, et $SH = k - z$. Est autem SM ad SH ut attractio $\frac{1}{\overline{SM}^2}$ corporis S versus particulam M in directione SM ad ejusdem corporis attractionem in directione SH, quæ proinde erit $\frac{SH}{\overline{SM}^3}$: sed est $SR = HL$, et $\overline{SM}^2 = \overline{SR}^2 + \overline{MR}^2 = \overline{SR}^2 + \overline{SH}^2 + \overline{ML}^2$; unde fit $\frac{SH}{\overline{SM}^3} = \frac{SH}{\overline{\overline{HL}^2 + \overline{SH}^2 + \overline{ML}^2}^{\frac{3}{2}}}$, et ductâ mn parallelâ ad MN, vis qua corpus S attrahitur ad arcus quàm minimos Mm, Nn, exponitur per $\frac{SH \times 2Mm}{\overline{SM}^3} = SH \times 2Mm \times \overline{\overline{HL}^2 + \overline{SH}^2 + \overline{ML}^2}^{-\frac{3}{2}}$. Est autem $\overline{HL}^2 + \overline{SH}^2 + \overline{ML}^2 = kk - 2kz + zz + hh - 2hx + rr$, hincque ponendo $kk + hh = ll$, $\overline{\overline{HL}^2 + \overline{SH}^2 = \overline{ML}^2}^{-\frac{3}{2}} = \frac{1}{l^3} + \frac{3kz}{l^5} + \frac{3hx}{l^5} - \frac{3rr}{2l^5} - \frac{3zz}{2l^5} + \frac{15kkzz}{2l^7} + \frac{15khzx}{2l^7} + \frac{15hhxx}{2l^7}$, neglectis terminis ulterioribus ob longinquitatem quam supponimus corporis S. Quarè, si scribatur d pro circumferentiâ IMKN, gravitas corporis S ad totam illam circumferentiam secundum SH, sive fluens fluxionis $SH \times 2Mm \times \overline{\overline{HL}^2 + \overline{SH}^2 + \overline{ML}^2}^{-\frac{3}{2}}$ evadit $\overline{k - z} \times d$ in $\frac{1}{l^3} + \frac{3kz}{l^5} - \frac{3rr}{2l^5} - \frac{3zz}{2l^5} + \frac{15kkzz}{2l^7}$

$\frac{15kkzz}{2l^7} + \frac{15hhrr}{4l^7}$. Simili modo obtinebitur gravitas ejusdem corporis S secundum SR. *Q. E. I.*

LEMMA II.

Corporis longinqui gravitatem ad Sphæroidem oblatam determinare.

Retentis iis quæ sunt in lemmate superiori demonstrata; esto C centrum sphæroidis, cujus æquatori parallelus sit circulus IMK. Sphæroidis hujus semiaxis major sit a, semiaxis minor b, eorum differentia c, quam exiguam esse suppono; et dicatur D circumferentia æquatoris. Centro C et radio æquali semiaxi minori describi concipiatur circulus qui secet IK in i, eritque gravitas in directione SD, qua urgetur corpus S versus materiam sitam inter circumferentiam IMKN et circumferentiam centro X et radio Xi descriptam, æqualis gravitati in lemmate præcedenti definitæ ductæ in rectam Ii. Sed est Ii . c :: IX . a, atque d . D :: IX . a; unde I$i \times d$. D $\times c$ :: $\overline{IX}^2 . aa$, hoc est, ex naturâ ellipseos, ob CX $= z$, et IX $= r$, I$i \times d$. D $\times c$ :: $bb - zz$. bb, adeoque I$i \times d = \frac{D \times c}{bb} \times \overline{bb - zz}$, atque $rr = aa - \frac{aazz}{bb}$; scribi autem potest in sequenti calculo $bb - zz$ pro rr ob parvitatem differentiæ semiaxium in quam omnes termini ducuntur. Gravitas igitur corporis S in materiam inter circumferentias supradictas consistentem exprimetur per $\frac{D \times c}{bb} \times \overline{bb - zz} \times \overline{k - z}$ in $\frac{1}{l^3} + \frac{3kz}{l^5} - \frac{3bb}{2l^5} - \frac{15zz}{4l^5} + \frac{15bbbb}{4l^7} + \frac{45kkzz}{4l^7}$. Et si addatur gravitas in similem materiam ex

ex alterâ parte centri C ad æqualem à centro diſtantiam, quia tunc CX ſive z evadit negativa, gravitas corporis S in hanc duplicem materiam erit $\frac{D \times c}{bb} \times \overline{bb - zz}$ in $\frac{2k}{l^3} - \frac{6kzz}{l^5} - \frac{3kbb}{l^5} + \frac{15k^3zz}{l^7} + \frac{15hhkbb}{2l^7} - \frac{15hhkzz}{2l^7}$. Ducatur jam gravitas hæc in $\dot{z}$, et ſumptâ gravitatum omnium ſummâ, factâ $z = b$, gravitatio tota corporis S in totam materiam globo interiori ſuperiorem ſecundum directionem SD æquatori perpendicularem prodit $D \times c \times \overline{\frac{4kb}{3l^3} - \frac{4kb^3}{5l^5} + \frac{2khhb^3}{l^7}}$. Simili ratiocinio gravitatio corporis S in eamdem materiam ſecundum directionem SR æquatori parallelam invenitur æqualis $D \times c \times \overline{\frac{4hb}{3l^3} + \frac{2hb^3}{5l^5} - \frac{2hkkb^3}{l^7}}$. Tum ſi addatur gravitatio corporis S in globum interiorem, ex unâ parte ſcilicet $\frac{2b^3kD}{3al^3}$, et ex alterâ $\frac{2b^3hD}{3al^3}$, habebitur gravitas corporis S in totum ſphæroidem. *Q. E. I.*

Coroll.

Igitur gravitas corporis S ſecundum SD eſt ad ejuſdem gravitatem ſecundum SR ſive DC in materiam ſphæroidis globo interiori incumbentem ut $\frac{2k}{3} - \frac{2kb^2}{5l^2} + \frac{khhb^2}{l^4}$ ad $\frac{2h}{3} + \frac{hb^2}{5l^2} - \frac{hkkb^2}{l^4}$, adeoque ſi gravitas prior exponatur per k, poſterior exprimetur per $h - \frac{3hb^2}{5l^2}$ quamproximè. Unde cum ſit DC $= h$, patet gravitatem corporis S in ſphæroidem oblatam non tendere

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ad centrum C, ſed ad punctum c rectæ DC in plano æquatoris jacentis vicinius puncto D.

PROPOSITIO I.

PROBLEMA.

Vires determinare quibus perturbatur motus Satellitis circa Primarium ſuum revolventis.

Exhibeat jam ſphærois prædicta planetam quemvis figurâ hac donatum, et corpus S ſatellitem circa planetam tanquàm primarium gyrantem. Quantitas materiæ globo ſphæroidis interiori incumbentis æqualis eſt $\frac{4bbcD}{3a}$ ſive $\frac{4bcD}{3}$ proximè, et ſi materia illa locaretur in centro ſphæroidis C, attraheret ſatellitem S ſecundum SC vi $\frac{4bcD}{3l^2}$, quæ reducta ad directionem SD fit $\frac{4bckD}{3l^3}$, et ad directionem DC fit $\frac{4bchD}{3l^3}$. Cum igitur vis $\frac{4bcD}{3l^2}$ non turbat motum ſatellitis, utpote quæ tendat ad centrum motûs et quadrato diſtantiæ ab eodem centro ſit reciprocè proportionalis, vires illæ $\frac{4bckD}{3l^3}$, $\frac{4bchD}{3l^3}$, in quas reſolvitur, etiam motum non turbabunt. Itaque ex vi $D \times c \times \overline{\frac{4kb}{3l^3} - \frac{4kb^3}{5l^5} + \frac{2khhb^3}{l^7}}$ auferatur vis $\frac{4bckD}{3l^3}$, et ex vi $D \times c \times \overline{\frac{4hb}{3l^3} + \frac{2hb^3}{5l^5} - \frac{2hkkb^3}{l^7}}$ auferatur $\frac{4bchD}{3l^3}$, et remanebunt vires $D \times c \times \overline{-\frac{4kb^3}{5l^5} + \frac{2khhb^3}{l^7}}$, $D \times c \times \overline{\frac{2hb^3}{5l^5} - \frac{2hkkb^3}{l^7}}$, motuum ſatellitis S perturbatrices. Deſignetur vis $D \times c \times$

$\overline{\frac{2hb^3}{5l^5} - \frac{2hhkb^3}{l^7}}$ per rectam Sr (*Fig.* 2.) ac resolvatur in vim Sq tendentem ad centrum planetæ primarii C et ob triangula similia Srq, SDC, æqualem $D \times c \times \overline{\frac{2b^3}{5l^4} - \frac{2kkb^3}{l^6}}$, existentibus ut priùs, $SD = k$, $DC = h$, $SC = l$; et in vim rq rectæ SD parallelam et æqualem $D \times c \times \overline{\frac{2kb^3}{5l^5} - \frac{2k^3b^3}{l^7}}$; atque hæc vis posterior subducta ex vi $D \times c \times \overline{- \frac{4kb^3}{5l^5} + \frac{2khhb^3}{l^7}}$ relinquet $D \times c \times \frac{4kb^3}{5l^5}$ pro vi perturbatrice in directione SD. Unde cum massa tota planetæ sit $\frac{2abD}{3}$, gravitas satellitis tota in planetam erit $\frac{2abD}{3l^2}$ proximé, vel etiam $\frac{2bbD}{3l^2}$, et hæc gravitas est ad vim $D \times c \times \frac{4kb^3}{5l^5}$ ut 1 ad $\frac{6kbc}{5l^3}$.

Deinde vis illius $D \times c \times \frac{4kb^3}{5l^5}$ secundum SD pars ea quæ agit in directione SC est $D \times c \times \frac{4kkb^3}{5l^6}$, quæ addita vi S$q$ dat $D \times c \times \overline{\frac{2b^3}{5l^4} - \frac{6kkb^3}{5l^6}}$ vim perturbatricem tendentem ad centrum planetæ primarii, atque hæc vis est ad satellitis gravitatem $\frac{2bbD}{3l^2}$ in primarium ut $\frac{3bc}{5l^2} - \frac{9kkbc}{5l^4}$ ad 1. *Q. E. I.*

COROLL.

Designet CK (*Fig.* 3.) lineam intersectionis planorum æquatoris planetæ et orbitæ satellitis, et resolvatur vis $SD = \frac{6kbc}{5l^3}$, quæ agit perpendiculariter ad

 planum

planum æquatoris, in vim DR perpendicularem ad planum orbitæ satellitis, et in vim SR jacentem in eodem plano. Producatur SR donec occurrat CK in K, eritque SK normalis ad CK, et planum SDK normale ad planum orbis satellitis; ac propterea ob similia triangula SDK, SRD, si m denotet sinum ad radium 1 et n cosinum anguli SKD, inclinationis scilicet orbitæ satellitis ad æquatorem planetæ, erit $DR = SD \times n = \frac{6kbcn}{5l^3}$, et $SR = SD \times m = \frac{6kbcm}{5l^3}$, existente 1 gravitate totâ satellitis in primarium suum. Jam quoniam vis SR jacet in plano orbitæ satellitis, hujus plani situm non mutat; accelerat quidem vel retardat motum satellitis revolventis, sed hæc acceleratio vel retardatio ob brevitatem temporis ad quantitatem sensibilem non exurgit: vis DR eidem plano perpendicularis continuò mutat ejus situm, et motum nodi generat, quem sequenti propositione definiemus.

PROPOSITIO II.

PROBLEMA.

Invenire motum nodi ex prædictâ causâ oriundum.

Per motum nodi in hac propositione intelligo motum intersectionis planorum æquatoris planetæ et orbitæ satellitis; orbitam autem satellitis quamproximé circularem suppono. Esto S locus satellitis in orbe suo SN cujus centrum C, (*Fig.* 4.) SF arcus centro C descriptus perpendicularis in circulum æquatoris planetæ FN; SB arcus eodem centro descriptus perpendicularis ad orbem SN, atque in SB sumatur lineola Sr æqualis duplo spatio, quod satelles percurrere posset impellente vi DR in Coroll. præced.

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determinatâ, quo tempore in orbe suo describeret arcum quàm minimum pS: per puncta r, p, describatur centro C circulus rpn secans equatorem in n, qui exhibebit situm orbitæ satellitis post illam particulam temporis, nodo N translato in n. Agantur SC, CN, et SH perpendicularis in lineam nodorum CN, et Nm perpendicularis in rpn. Jam cum sint lineolæ Sr, Nm, ut sinus arcuum Sp, SN, erit $Sp . Sr :: SH . Nm$; deinde in triangulo rectangulo Nmn habetur $m . 1 :: Nm . Nn$; unde per compositionem rationum $Sp \times m . Sr :: SH . Nn = \frac{Sr \times SH}{Sp \times m}$: dato igitur arcu Sp, est Nn sive motus nodi ut $Sr \times SH$. In triangulo sphærico rectangulo SFN est sinus anguli N, hoc est, anguli inclinationis orbitæ satellitis ad æquatorem planetæ, ad sinum arcûs SF, ut radius ad sinum arcûs SN, id est, $m . \frac{k}{l} :: 1 . SH$, adeoque $\frac{k}{l} = m \times SH$; est igitur $\frac{k}{l}$ ut SH. Vis autem Sr per Coroll. Prop. præced. est ut $\frac{k}{l}$, adeoque ut SH; quamobrem est $Sr \times SH$, proindeque et Nn, ut $\overline{SH}^2$, hoc est, motus horarius nodi vi præfatâ genitus est in duplicatâ ratione distantiæ satellitis à nodo. Et quoniam summa omnium $\overline{SH}^2$, quo tempore satelles periodum suam absolvit, est dimidium summæ totidem $\overline{SC}^2$, ideò motus periodicus est subduplus ejus qui, si satelles in declinatione suâ maximâ ab æquatore planetæ continuò perstaret, eodem tempore generari posset. Sit igitur satelles in maximâ suâ declinatione sive in quadraturâ cum nodo, eritque SN quadrans circuli, et Nm mensura anguli Npm sive Spr, eritque in hoc casu Nn sive motus horarius nodi ad Nm, hoc est, ad angulum Spr, ut 1 ad m;

eſt autem angulus Spr ad duplum angulum, quem ſubtendit ſinus verſus arcûs Sp ſatellitis gravitate in primarium eodem tempore deſcripti, id eſt, ad angulum SCp qui eſt motus horarius ſatellitis circa primarium, ut vis Sr ad gravitatem ſatellitis in primarium, hoc eſt (per Coroll. Prop. I.), ut $\frac{6kbcn}{5l^3}$ ad 1, ſive, quia eſt in hoc caſu $\frac{k}{l} = m$, ut $\frac{6bcmn}{5l^2}$ ad 1. Unde conjunctis rationibus eſt motus horarius nodi ad motum horarium ſatellitis ut $\frac{6bcn}{5l^2}$ ad 1; et ſi S denotet tempus periodicum ſolis apparens, et L tempus periodicum ſatellitis circa primarium ſuum, cum ſit motus horarius ſatellitis ad motum horarium ſolis ut S ad L, erit motus horarius nodi ad motum horarium ſolis ut $\frac{6bcn}{5l^2} \times \frac{S}{L}$ ad 1, et in eadem ratione erit motus nodi annuus ad motum ſolis annuum, hoc eſt, ad 360°. Quarè, ſi ſatelles maneret toto anno in maximâ ſuâ declinatione ab æquatore primarii, vis prædicta ex figurâ ſphæroïdicâ planetæ primarii proveniens generaret eodem tempore motum nodi æqualem $\frac{6bcn}{5l^2} \times \frac{S}{L} \times 360°$, et ex ſupradictis motus verus nodi annuus erit hujus ſubduplus, nempe $\frac{3bcn}{5l^2} \times \frac{S}{L} \times 360°$. *Q. E. I.*

COROLL.

Si computatio inſtituatur pro lunâ, aſſumendo mediocrem ejus orbitæ inclinationem ad æquatorem terreſtrem, erit n coſinus anguli 23° 28′½; et poſito ſemiaxi terræ $b = 1$, erit diſtantia lunæ à centro terræ mediocris $l = 60$ circiter, indeque in hypotheſi quod ſit

ſit differentia ſemiaxium $c = \frac{1}{229}$, erit $\frac{3bcn}{5l^2} \times \frac{S}{L} \times 360^\circ = 11''\frac{1}{2}$; et ſi fuerit $c = \frac{1}{177}$, manente terrâ uniformiter denſâ, erit ille motus $= 15''$. Hic erit motus nodorum annuus lunæ regreſſivus in plano æquatoris terreſtris, qui reductus ad eclipticam, uti poſteà docebitur, pro vario nodorum ſitu evadet multò velocior.

Notabilis multò magis erit motus interſectionis orbitarum ſatellitum Jovis in plano æquatoris Jovialis; et computabitur ſatis accuratè per formulam ſuprà traditam, modò ſatelles non ſit Jovi nimis vicinus. Sic pro ſatellite extimo erit $L = 16^d\ 16^h\ 32'$, $b = 1$, $l = 25{,}299$ circiter, ſemiaxium Jovis differentia $c = \frac{1}{13}$; et poſitâ orbis hujus ſatellitis inclinatione ad æquatorem Jovis æquali 3°, erit n coſinus hujus inclinationis, atque inde prodibit $\frac{3bcn}{5l^2} \times \frac{S}{L} \times 360^\circ = 34'$ circiter, motus ſcilicet nodorum annuus ſatellitis quarti in plano æquatoris Jovis in antecedentia. Si minùs vel magìs inclinatur orbis ad Jovis æquatorem, augeri vel minui debet hic motus in ratione coſinûs hujus inclinationis.

Cæterùm patet motum hunc nodorum in plano æquatoris planetæ primarii, æſtimando diſtantiam ſatellitis in ſemidiametris primarii, generatìm eſſe, dato tempore, in ratione compoſitâ, ex ratione directâ differentiæ ſemiaxium planetæ et coſinûs inclinationis orbis ſatellitis ad planetæ æquatorem, conjunctìm; et ex ratione inverſâ temporis periodici ſatellitis et quadrati diſtantiæ ſatellitis à centro planetæ, item conjunctìm.

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PROPOSITIO III.

PROBLEMA.

Motum nodorum Lunæ ſupra determinatum ad Eclipticam reducere.

Sunto NAD (*Fig.* 5.) æquator, AGE ecliptica ſecans æquatorem in A, E æquinoctium vernum, A autumnale, LGN orbis lunæ ſecans eclipticam in G et æquatorem in N, LD circulus maximus perpendicularis in æquatorem; et ſunto DN, LN, quadrantes circuli. Tempore dato vi prædictâ transferratur interſectio N in n, et deſcribatur circulus Lgn exhibens ſitum orbis lunaris poſt illud tempus, ſecetque eclipticam in g. Ut autem interſectiones N et G ſine verborum ambagibus diſtinguantur, priorem in poſterum vocabo *Nodum Æquatorium*, poſteriorem *Nodum Eclipticum*. Ductis itaque Nm, Gd, perpendicularibus in orbem lunæ, eſt Nn : Nm :: 1 : ſin. GNA, et Nm : Gd :: 1 : ſin. LG, itemque Gd : Gg :: ſin. Ggd : 1; unde conjunctis rationibus provenit Nn : Gg :: ſin. Ggd : ſin. GNA × ſin. LG, adeoque $\mathrm{G}g = \mathrm{N}n \times \frac{\text{ſin. GNA} \times \text{ſin. LG}}{\text{ſin. G}gd}$. Scribantur s pro ſinu et t pro coſinu anguli Ggd, inclinationis ſcilicet orbitæ lunaris ad eclipticam, ad radium 1, v pro ſinu et u pro coſinu arcûs EG, p pro ſinu et q pro coſinu obliquitatis eclipticæ; atque per reſolutionem trianguli ſphærici GAN, habebitur coſ. GNA $= n = qt + psu$, indeque ſin. GNA $= \sqrt{1 - qqtt - 2pqstu - p^2s^2u^2}$; ſed ſcribi poteſt 1 pro t, et rejici terminus $p^2s^2u^2$ ob exiguitatem ſinûs s anguli

5°

$5^{\circ}\ 8'\frac{1}{2}$, proindeque erit ſin. GNA $= \sqrt{pp - 2pqsu}$; prætereà eſt ſin. GNA : ſin. GA ſive v :: ſin. GAN ſive p : ſin. GN, ideoque ſin. GN ſive coſ. LG $= \frac{pv}{\text{ſin. GNA}}$, et ſin. LG $= u - \frac{qs.vv}{p}$, ac ſin. GNA $\times$ ſin. LG $= pu - qs$ quamproximé. Quarè fit Gg $=$ N$n \times \frac{pu - qs}{s}$, atque hic eſt motus nodorum lunarium tempore dato in plano eclipticæ: quod ſi tempus illud datum ſit annus ſolaris, habetur Nn $= \frac{3bcn}{5l^2} \times \frac{S}{L} \times 360^{\circ}$, unde motus ille eclipticus nodorum annuus, nullâ habitâ ratione mutationis ſitûs nodorum ex aliâ cauſâ per id temporis factæ, fiet $\frac{3bc}{5l^2} \times \overline{qt + psu} \times \frac{pu - qs}{s} \times \frac{S}{L} \times 360^{\circ}$, vel etiam $\frac{3bcq}{5l^2} \times \frac{pu - qs}{s} \times \frac{S}{L} \times 360^{\circ}$ proximé. *Q. E. I.*

Quo motum nodi lunaris in hac propoſitione ad eclipticam reduximus, eodem prorſùs ratiocinio motus nodi ſatellitis cujuſvis ad orbitam planetæ primarii reducetur.

COROLL. I.

Exinde liquet nullum eſſe hunc motum nodi, ubi ſin. LG $= 0$, vel etiam ubi $pu = qs$, quod contingit ubi orbitæ lunaris arcus GN eclipticam et æquatorem æqualis eſt 90°, ſive ubi nodi lunares verſantur in punctis declinationis lunaris maximæ, ſive ubi arcus AG, cujus coſinus eſt u, evadit æqualis $78^{\circ}\ 5'$, id eſt, ubi nodus aſcendens lunæ verſatur in $11^{\circ}\ 55'$ Cancri, vel $18^{\circ}\ 5'$ Sagittarii. Eritque progreſſivus hic motus, id eſt, fiet ſecundum ſeriem ſignorum, dum nodus aſcendens lunæ tranſit retrocedendo ab

18° 5′ Sagittarii ad 11° 55′ Cancri, regreſſivus autem in reliquâ parte revolutionis; et maximus evadit motus regreſſivus, ubi $u = -1$, id eſt, ubi nodus aſcendens verſatur in principio Arietis; et maximus progreſſivus, ubi $u = 1$, id eſt, ubi idem nodus occupat initium Libræ. Itaque cùm motus ille nodorum annuus, de quo hîc agitur, univerſaliter ſit æqualis $\frac{3bcq}{5l^2} \times \frac{pu - qs}{s} \times \frac{S}{L} \times 360°$, hoc eſt, per Coroll. Prop. 2. æqualis $11''\frac{1}{2} \times \frac{pu - qs}{s}$ vel $15'' \times \frac{pu - qs}{s}$ prout differentia ſemiaxium terræ fuerit $\frac{1}{229}$ vel $\frac{1}{177}$, exiſtentibus ſcilicet p ſinu et q coſinu anguli 23° 28′$\frac{1}{2}$, atque s ſinu anguli 5° 8′$\frac{1}{2}$; eo anno, in cujus medio circiter nodus lunæ aſcendens tenuerit principium Arietis, motus nodorum regreſſivus, qui et maximus, erit 1′ 2″ vel 1′ 20″; ubi verò idem nodus ſubierit ſignum Libræ, motus maximus progreſſivus erit 41″ vel 53″. In aliis nodorum poſitionibus eodem modo computabitur.

Coroll. II.

Si deſideretur exceſſus regreſſûs nodi ſupra progreſſum in integrâ nodi revolutione, ſequenti ratione inveſtigabitur. Jungantur equinoctia diametro EA, in quam demittatur perpendiculum GK, et ſumpto arcu Gb quem deſcribit nodus eclipticus G quo tempore nodus equatorius N deſcribit arcum Nn, ducatur bc perpendicularis in GK. Per hanc propoſitionem eſt $Gg \,.\, Nn :: \frac{pu - qs}{s} \,.\, 1$, ſive, quia eſt $1 \,.\, u :: Gb \,.\, Gc$, fit $Gg \,.\, Nn :: \frac{p \times Gc}{s} - q \times Gb \,.\, Gb$; adeoque ſumma omnium Gg erit ad ſummam omnium

nium Nn, hoc eſt, motus nodi ecliptici in integrâ ſui revolutione erit ad motum nodi æquatorii eodem tempore factum, ut ſumma omnium in circulo quantitatum $\frac{p \times Gc}{s} - q \times Gb$ ad ſummam totidem arcuum Gb, hoc eſt, ut $-q$ ad 1. Signum autem $-$ denotat motum fieri in antecedentia ſive regreſſum nodi excedere ejuſdem progreſſum. Unde cum motus nodi æquatorii N ſit $11''\frac{1}{2}$ vel $15''$ quo tempore nodus eclipticus deſcribit $19^\circ\ 20'\frac{1}{2}$, motus ille nodi æquatorii tempore nodi ecliptici periodico evadit $11''\frac{1}{2} \times \frac{360^\circ}{19^\circ\ 20\frac{1}{2}} \doteq 3'\ 34''$ vel $15'' \times \frac{360^\circ}{19^\circ\ 20\frac{1}{2}} = 4'\ 39''$; quo pacto prodit motus nodi ecliptici præfatus æqualis $q \times 3'\ 34''$ vel $q \times 4'\ 39''$, proindeque *eſt radius ad coſinum obliquitatis eclipticæ ut* $3'\ 34''$ *vel* $4'\ 39''$ *ad motum quæſitum*, nempe $3'\ 16''$, exiſtente $\frac{1}{229}$ differentiâ axium terræ, vel $4'\ 16''$ eâ exiſtente $\frac{1}{177}$: atque hic eſt exceſſus regreſſûs nodi ſupra progreſſum in integrâ nodi revolutione vi prædictâ genitus. Exceſſu igitur hoc minuatur motus nodi lunaris periodicus 360°, et remanebit motus ille quem generat vis ſolis.

PROPOSITIO IV.

PROBLEMA.

Variationem inclinationis orbis lunaris ad planum eclipticæ ex figurâ terræ ſpheroidicâ ortam determinare.

Eſto ANH (*Fig.* 6.) æquator, AG ecliptica, et A punctum æquinoctii autumnalis: ſit NGRM orbis lunæ ſecans eclipticam in G et æquatorem in N, in

quo ſumantur arcus NL, GR, æquales quadrantibus circuli. Jam ſi nodus æquatorius N per temporis particulam vi prædictâ transferri intelligatur in n, et per punctum L deſcribatur circulus n L r, exhibebit hic ſitum orbis lunæ poſt tempus elapſum, et ſi in eumdem demittantur perpendicula Nm et Rr, poſterius Rr deſignabit variationem inclinationis orbitæ lunaris ad eclipticam eodem tempore genitam. Eſt autem Nn : Nm :: 1 : m, itemque Nm : Rr :: 1 : ſin. LR; ſed ob NL = GR, eſt NG = LR; unde conjunctis rationibus eſt Nn : Rr :: 1 : m × ſin. NG; ex quo patet variationem inclinationis momentaneam eſſe proportionalem ſinui diſtantiæ nodi lunaris ecliptici à nodo æquatorio. Ad diametrum NM demittatur perpendiculum GK, et exiſtente Gb decremento arcûs NG facto quo tempore nodus æquatorius N deſcribit arcum Nn, agatur bk parallela ipſi GK, eritque 1 : GK ſive ſin. NG :: Gb . Kk; proindeque jam erit Nn : Rr :: Gb : m × Kk, adeoque ſumma omnium variationum Rr, quo tempore nodus eclipticus G deſcripſit arcum MG, genitarum erit ad ſummam totidem motuum Nn, hoc eſt, ad motum nodi æquatorii N eodem tempore factum, ut ſumma omnium Kk ducta in m, ad ſummam totidem arcuum Gb, id eſt, ut m × MK ad MG. Sit NH motus nodi N tempore revolutionis nodi G ab uno equinoctio ad alterum, eritque variatio inclinationis eodem tempore genita, hoc eſt, variatio tota æqualis $\frac{2m \times \text{NH}}{\text{MGN}}$. Unde cùm $\frac{\text{NH}}{\text{MGN}}$ exprimat rationem motûs nodi æquatorii ad motum nodi ecliptici, prodit theorema ſequens: *Eſt motus nodi lunaris ecliptici ad motum nodi æquatorii, ut ſinus duplicatus inclinationis medio-*

cri

cris orbitæ lunaris ad æquatorem, ad ſinum variationis totius inclinationis ejuſdem orbitæ ad eclipticam.

In hoc computo inclinationem mediocrem orbis lunaris ad æquatorem, nempe $23° 28'\frac{1}{2}$, uſurpo, cum in revolutione nodi tantum ex unâ parte augetur, quantum ex alterâ minuitur, et omnes minutias hîc expendere ſupervacaneum foret. Motus autem nodi lunaris ecliptici eſt ad motum nodi lunaris æquatorii ut $19° 20'\frac{1}{2}$ ad $11''\frac{1}{2}$ vel $15''$, ſive ut 6055 vel 4642 ad 1, unde per theorema ſupra traditum prodit variatio inclinationis tota æqualis $27''$ vel $35''$, prout differentia axium terræ ſtatuitur $\frac{1}{229}$ vel $\frac{1}{177}$. Hac igitur quantitate augetur inclinatio orbis lunaris ad eclipticam in tranſitu nodi aſcendentis lunæ ab æquinoctio vernali ad autumnale, et tantumdem minuitur in alterâ medietate revolutionis nodi. In loco quolibet G inter æquinoctia variatio inclinationis eſt ad variationem totam ut ſinus verſus arcûs MG ad diametrum, ut patet; ſive differentia inter ſemiſſem variationis totius et variationem quæſitam eſt ad ipſam ſemiſſem variationis totius ut coſinus arcûs MG ad radium, hoc eſt, ut $u - \frac{qsvv}{p}$ ad 1. *Q. E. I.*

PROPOSITIO V.

Problema.

Motum apſidum in orbe ſatellitis quamproximè circulari, quatenùs ex figurâ planetæ primarii ſphæroidicâ oritur, inveſtigare.

Per propoſitionem primam vis perturbatrix, quâ trahitur ſatelles ad centrum planetæ primarii, eſt ad

 ſatellitis

ſatellitis gravitatem in ipſum primarium, ut $\frac{3bc}{5l^2} - \frac{9kkbc}{5l^4}$ ad 1, ſive, quia per Prop. 2. eſt $\frac{k}{l} = m \times SH$ (*Fig.* 4.) ponendo ſcilicet m pro ſinu inclinationis orbitæ ſatellitis ad æquatorem primarii, et ſcribendo y pro SH, ut $\frac{3bc}{5l^2} \times \overline{1 - 3m^2y^2}$ ad 1; et ſumma harum virium in totâ circumferentiâ cujus radius eſt 1, eſt ad gravitatem ſatellitis toties ſumptam ut $\frac{3bc}{5l^2} \times \overline{1 - \frac{3m^2}{2}}$ ad 1. Vis igitur mediocris, quæ uniformiter agere in ſatellitem ſupponi poteſt, dum revolutionem ſuam in orbitâ propemodùm circulari abſolvit, eſt ad ejus gravitatem in primarium ut $\frac{3bc}{5l^2} \times \overline{1 - \frac{3m^2}{2}}$ ad 1; atque hac vi movebuntur apſides, ſi nulla habeatur ratio vis alterius quæ orbis radio eſt perpendicularis et per medietatem revolutionis ſatellitis in unum ſenſum tendit, per alteram medietatem in contrarium. Jam quia ex demonſtratis in hac et primâ propoſitione ſequitur gravitatem ſatellitis circa planetam, cujus figura eſt ſphærois oblata, revolventis in diſtantiâ l generaliter eſſe ad ejuſdem gravitatem in majori diſtantiâ L, ut $\frac{1}{l^2} + \frac{B}{l^4} \times \overline{1 - \frac{3m^2}{2}}$ ad $\frac{1}{L^2} + \frac{B}{L^4} \times \overline{1 - \frac{3m^2}{2}}$, exiſtente B quantitate datâ exigui valoris, ſive ut $\frac{1}{l^2}$ ad $\frac{1}{L^2} - \frac{B}{l^2L^2} \times \overline{1 - \frac{3m^2}{2}} + \frac{B}{L^4} \times \overline{1 - \frac{3m^2}{2}}$ quamproximè, ideò gravitas ſatellitis diminuitur in majori quam duplicatâ ratione diſtantiæ auctæ quoties m minor eſt quantitate $\sqrt{\frac{2}{3}}$, id eſt, ubi inclinatio orbitæ ſatellitis ad planetæ æquatorem non attingit 54° 44′.

44′; diminuitur autem in minori ratione, quoties eſt m major quàm $\sqrt{\frac{2}{3}}$, id eſt, ubi illa inclinatio ſuperat 54° 44′; adeoque in priore caſu progrediuntur apſides orbis ſatellitis, in poſteriori regrediuntur. Quantitas autem hujus progreſſûs vel regreſſûs ſic innoteſcet.

Per exemplum tertium prop. 45. lib. 1. *Princ. Math. Newt.* ſi vi centripetæ, quæ eſt ut $\frac{1}{l^2}$, addatur vis altera ut $\frac{e}{l^4}$, hoc eſt, quæ ſit ad vim centripetam $\frac{1}{l^2}$ ut $\frac{e}{l^2}$ ad 1, angulus revolutionis ab apſide unâ ad eamdem erit $360° \sqrt{\frac{1+e}{1-e}}$ vel $\frac{360°}{1-e}$ quamproximé, exiſtente e quantitate valdé minutâ. Porrò cum ſit motus ſatellitis in orbitâ ſuâ revolventis ad motum apſidis ut $\frac{360°}{1-e}$ ad $\frac{360°}{1-e} - 360°$, hoc eſt, ut 1 ad e, erit motus apſidis tempore revolutionis ſatellitis ad ſidera æqualis $360° \times e$, et hic motus apſidis erit ad ejuſdem motum tempore alio quovis dato ut tempus periodicum ſatellitis ad tempus datum. Eſt autem in hac noſtrâ propoſitione $e = \frac{3bc}{5l^2} \times \overline{1 - \frac{3m^2}{2}}$; unde datur motus apſidum quæſitus. *Q. E. I.*

COROLL.

Si ad lunam referatur hæc determinatio, habebuntur $b = 1$, $l = 60$, $m =$ ſinui anguli 23° 28′$\frac{1}{2}$, et ſi fuerit $c = \frac{1}{229}$, erit $e = \frac{1}{1803203}$, atque motus apogæi lunæ ſpatio centum annorum æqualis 16′ proximé in conſequentia; ſi fuerit $c = \frac{1}{177}$, erit $e = \frac{1}{1393742}$, et motus apogæi æqualis 20′, 7. Hac igitur quantitate minuendus eſt motus medius apogæi lunæ

prout

prout obſervationibus determinatur, ut habeatur motus ille quem generat vis ſolis.

Pro quarto autem Jovis ſatellite, erunt $b = 1$, $l = 25{,}299$, $c = \frac{1}{13}$, $m =$ ſinui anguli 3°, $e = \frac{1}{13924{,}7}$; hincque motus apſidis ſpatio unius anni ſolaris prodit $33'$, 95 vel ferè $34'$ in conſequentia, qui tempore annorum decem fit $5^\circ\ 40'$. Inſuper autem notandum eſt vi ſolis perturbari motum ſatellitis ſimili modo quo perturbatur motus lunæ; ideoque, quoniam vis ſolis, quâ perturbatur motus lunæ eſt ad lunæ gravitatem in terram in duplicatâ ratione temporis periodici lunæ circa terram ad tempus periodicum terræ circa ſolem, hoc eſt, ut 1 ad 178,725; pariter vis ſolis, qua perturbatur motus ſatellitis Jovialis, eſt ad ipſius ſatellitis gravitatem in Jovem in duplicatâ ratione temporum periodicorum ſatellitis circa Jovem et Jovis circa ſolem, hoc eſt, ut 1 ad 67394,6: vires igitur, quibus perturbantur motus lunæ et ſatellitis, ſunt ad ſe invicem, relativé ad eorum gravitates in planetas ſuos primarios ut $\frac{1}{178{,}725}$ ad $\frac{1}{67394{,}6}$, ſive ut 37,708 ad 1. Unde cum viribus ſimilibus proportionales ſunt motus his viribus dato tempore geniti, ſi vis prior vel ejuſdem vis pars quælibet motum apſidis generat æqualem $40^\circ\ 40'\frac{1}{2}$ in orbe lunari annuatìm, vis poſterior vel ejuſdem pars ſimilis et proportionalis motum apſidis eodem tempore generabit æqualem $6'\frac{1}{2}$ in orbe ſatellitis, atque decem annorùm ſpatio $1^\circ\ 5'$ in conſequentia. Addatur $1^\circ\ 5'$ ad $5^\circ\ 40'$, et motus apſidum totus in orbe ſatellitis extimi Jovialis ex duabus prædictis cauſis oriundus ſpatio decem annorum erit $6^\circ\ 45'$ in conſequentia. Obſervationibus Aſtronomicis collegit Ill. *Bradleius* hunc motum tempore prædicto eſſe quaſi 6°; differentia illa qua-

liſcumque

5

liſcumque 45′ inter motum obſervatum et computatum actionibus ſatellitum interiorum debebit aſcribi.

Scholium.

Ex præcedentibus colligere licet motuum lunarium inæqualitates originem ſuam omnem non ducere ex vi ſolis, ſed earum partem aliquam deberi actioni Telluris quatenùs induitur figurâ ſphæroidicâ. Sufficiat hîc illarum computaſſe valorem, et legem, quâ generantur, demonſtraſſe: utrum autem hujuſmodi correctiones tales ſint ut tabulis Aſtronomicis inſcribi mereantur, dijudicent Aſtronomi.

Item manifeſtum eſt præter inæqualitates eas, quæ in motibus ſatellitum Jovialium ex vi ſolis et actionibus ſatellitum in ſe invicem naſcuntur, oriri alias ex figurâ Jovis ſphæroidicâ ità notabiles ut Obſervationes Aſtronomicas continuò afficere debeant.

De Variatione motûs Terræ diurni.

Si terra globus eſſet omninò ſphæricus quicumque foret revolutionis axis, manente eâdem in globo motûs quantitate, eadem maneret rotationis velocitas: ſecùs autem eſt, ubi ob vires ſolis et lunæ terra induit formam ſphæroidis oblongæ per aquarum aſcenſum. Hîc enim non conſidero figuram telluris oblatam ob materiæ in æquatore redundantiam, ſed ſphæricam ſuppono niſi quatenùs per aquarum elevationem et depreſſionem in ſphæroidicam mutatur. Jam verò in ſphæroide hujuſmodi, quamvis eadem maneat motûs quantitas, mutatâ inclinatione axis tranſverſi ad axem revolutionis, mutabitur revolutionis velocitas, uti ſatis manifeſtum eſt: cùm autem axis

tranſverſus tranſit ſemper per ſolem vel lunam, ſingulis momentis mutabit ſitum ſuum reſpectu axis revolutionis ob motum quo hi duo planetæ recedunt ab æquatore terreſtri et ad eum viciſsìm accedunt.

PROBLEMA.

Variationem motûs terræ diurni ex prædictâ cauſâ oriundam inveſtigare.

Exhibeat ſphærois oblonga ADCd (*Fig.* 7.) terram fluidam, cujus centrum T, AC axis tranſverſus jungens centra terræ et ſolis vel lunæ, Dd axis minor, EO diameter æquatoris, et XZ axis motûs diurni. Centro T et radio TD deſcribatur circulus BDd ſecans axem tranſverſum AC in B, et agatur BK perpendicularis in TE: tum ex quovis circuli puncto P ductâ PM ad axem XZ normali quæ ſecet TA in H, ſit Ppr circumferentia circuli quam punctum P rotatione ſuâ diurnâ deſcribit, ad cujus quodvis punctum p ducatur Tp et producatur donec occurrat ſuperficiei ſphæroidis in q; deinde demiſſâ pG perpendiculari in PM, et GF perpendiculari in TA, ſi per puncta AqC tranſire intelligatur ellipſis ellipſi ADC ſimilis et æqualis, erit ex naturâ curvæ, quia ſphærois noſtra parùm admodùm differt à ſphærâ, $pq = \mathrm{AB} \times \frac{\overline{\mathrm{TF}}^2}{\overline{\mathrm{TP}}^2}$ quamproximé. Jam deſignet U velocitatem particulæ in terræ æquatore revolventis motu diurno circum axem XZ ad diſtantiam ſemidiametri TP, eritque $\frac{\mathrm{U} \times \mathrm{PM}}{\mathrm{TP}}$ velocitas particulæ P circulum Ppr deſcribentis, et cum ſit $\mathrm{TF} = \frac{\overline{\mathrm{GM} - \mathrm{HM}} \times \mathrm{TK}}{\mathrm{TP}} + \mathrm{TH}$, erit

motus

motus totius lineolæ pq æqualis $pq \times \frac{U \times PM}{TP} = \frac{U \times AB \times PM}{\overline{TP}^3} \times \frac{\overline{\overline{GM} - \overline{HM} \times \overline{TK}^2}}{TP} + TH$, adeoque ſumma horum motuum in circuitu circuli Ppr, hoc eſt, motus ſuperficiei inter circulum Ppr et ſphæroidem in directione Tp contentæ, æquabitur circumferentiæ hujus circuli ductæ in $\frac{U \times AB \times PM}{\overline{TP}^3} \times \overline{\frac{\overline{TK}^2 \times \overline{PM}^2}{2\,\overline{TP}^2} + \frac{\overline{TK}^2 \times \overline{HM}^2}{\overline{TP}^2} - \frac{2TK \times HM \times TH}{TP} + \overline{TH}^2}$ ſive quia eſt HM . TM :: TK . BK, et TH . HM :: TP . TK, ſcribendo D pro circumferentiâ circuli BDd, æquabitur ille motus quantitati $\frac{U \times AB \times D}{2\,\overline{TP}^6} \times \overline{\overline{TK}^2 \times \overline{PM}^4 + 2\overline{BK}^2 \times \overline{TM}^2 \times \overline{PM}^2}$. Deinde horum motuum ſumma in toto circuitu globi collecta, hoc eſt, motus totius materiæ globo BDd incumbentis prodibit æqualis $\frac{U \times AB \times DD}{32} \times \frac{3\overline{TP}^2 - \overline{BK}^2}{\overline{TP}^2}$. Ubi planeta in plano æquatoris conſiſtit, fit BK $= 0$, et motus prædictus æqualis $\frac{U \times 3AB \times DD}{32}$. Motus autem globi QPR circa eundem axem eſt (uti facilé demonſtratur) $\frac{U \times TP \times DD}{16}$, adeoque motus terræ totius fit $\frac{U \times TP \times DD}{16} + \frac{U \times AB \times DD}{32} \times \frac{3\overline{TP}^2 - \overline{BK}^2}{\overline{TP}^2}$, qui cum idem ſemper manere debeat, denotet V velocitatem in ſuperficie æquatoris terreſtris ubi planeta verſatur in plano æquatoris, eritque $\frac{U \times TP \times DD}{16} + \frac{U \times 3AB \times DD}{32} =$

$\frac{U \times TP \times DD}{16} + \frac{U \times AB \times DD}{32} \times \frac{3\overline{TP}^2 - \overline{BK}^2}{\overline{TP}^2}$; unde ſcribendo 1 pro TP quatenùs eſt radius ad ſinum BK anguli BTK, habetur $V . U :: TP + \frac{3 AB}{2} - \frac{AB \times \overline{BK}^2}{2} . TP + \frac{3 AB}{2}$, indeque, quia minima eſt altitudo AB reſpectu ſemidiametri TP, $U - V . V :: AB \times \overline{BK}^2 . 2 TP$, et $U - V = V \times \frac{AB \times \overline{BK}^2}{2 TP}$: pro V autem patet ſcribi poſſe velocitatem angularem terræ mediocrem quia ab eâ differt quàm minimé et ducitur in quantitatem perexiguam $\frac{AB \times \overline{BK}^2}{2 TP}$, et quia tempora revolutionum terræ circa centrum ſuum ſint reciprocé ut motus angulares U, V, fiet differentia revolutionum terræ ubi planeta æquatorem tenet et ubi ab æquatore diſtat angulo BTK, æqualis $23^h\, 56' \times \frac{AB \times \overline{BK}^2}{2 TP}$. Quoniam igitur eſt acceleratio horaria ad motum terræ horarium mediocrem circa centrum ſuum ut $AB \times \overline{BK}^2$ ad 2 TP ſive (quia eſt ſinus p inclinationis eclipticæ ad æquatorem ad radium 1 ut ſinus BK ad ſinum diſtantiæ planetæ ab æquinoctio, quem ſinum dico K) ut $AB \times p^2 \times K^2$ ad 2 TP; adeoque acceleratio horaria rotationis terræ creſcit in ratione duplicatâ ſinûs diſtantiæ planetæ à puncto æquinoctii, et ſumma omnium illarum accelerationum, quo tempore tranſit planeta ab æquinoctio ad ſolſtitium, eſt ad ſummam totidem motuum horariorum mediocrium, hoc eſt, acceleratio tota eo tempore genita eſt ad tempus illud ut ſumma quantitatum omnium $AB \times p^2 \times K^2$ in circuli quadrante ad ſum-

mam

mam totidem $2 TP$, id eſt, quia ſumma omnium K^2 in circuli quadrante dimidium eſt ſummæ totidem quadratorum radii, ut $AB \times p^2$ ad $4 TP$. Quamobrèm, ſi denotet P quartam partem temporis planetæ periodici circa terram, erit acceleratio tota motûs terræ circum axem ſuum in tranſitu planetæ ab æquinoctio ad ſolſtitium genita æqualis $\frac{AB \times P \times p^2}{4 TP}$, atque eadem erit retardatio in tranſitu planetæ à ſolſtitio ad æquinoctium. Unde ſponte naſcitur hoc Theorema: *Eſt quadratum diametri ad quadratum ſinûs obliquitatis eclipticæ ut quarta pars temporis periodici ſolis vel lunæ ad tempus aliud*; deinde, *eſt ſemidiameter terræ ad differentiam ſemiaxium ut tempus mox inventum ad accelerationem quæſitam.*

Aſcenſus aquæ AB vi ſolis debitus eſt duorum pedum circiter, exiſtente ſemidiametro terræ mediocri $TP = 19615800$, unde prodit per theorema acceleratio terræ circa centrum ſuum gyrantis facta quo tempore incedit ſol ab æquinoctio ad ſolſtitium, æqualis $1''' \, 55^{iv}$ in partibus temporis; et ſi vi lunæ aſcendunt aquæ ad altitudinem octo pedum, acceleratio revolutionis terræ inde orta, quo tempore luna tranſit ab æquatore ad declinationem ſuam maximam, erit 34^{iv}: et ſumma harum accelerationum, quæ obtinet ubi hi duo planetæ in punctis ſolſtitialibus verſantur, cùm non ſuperet duo minuta tertia temporis cum ſemiſſe ſive 37 minuta tertia gradûs, vix obſervabilis erit. *Q. E. I.*

Cùm igitur tantilla ſit hujuſmodi variatio in hypotheſi ſphæricitatis terræ; qualis evaderet, terrâ exiſtente ſphæroide oblatâ, fruſtrà quis inquireret.

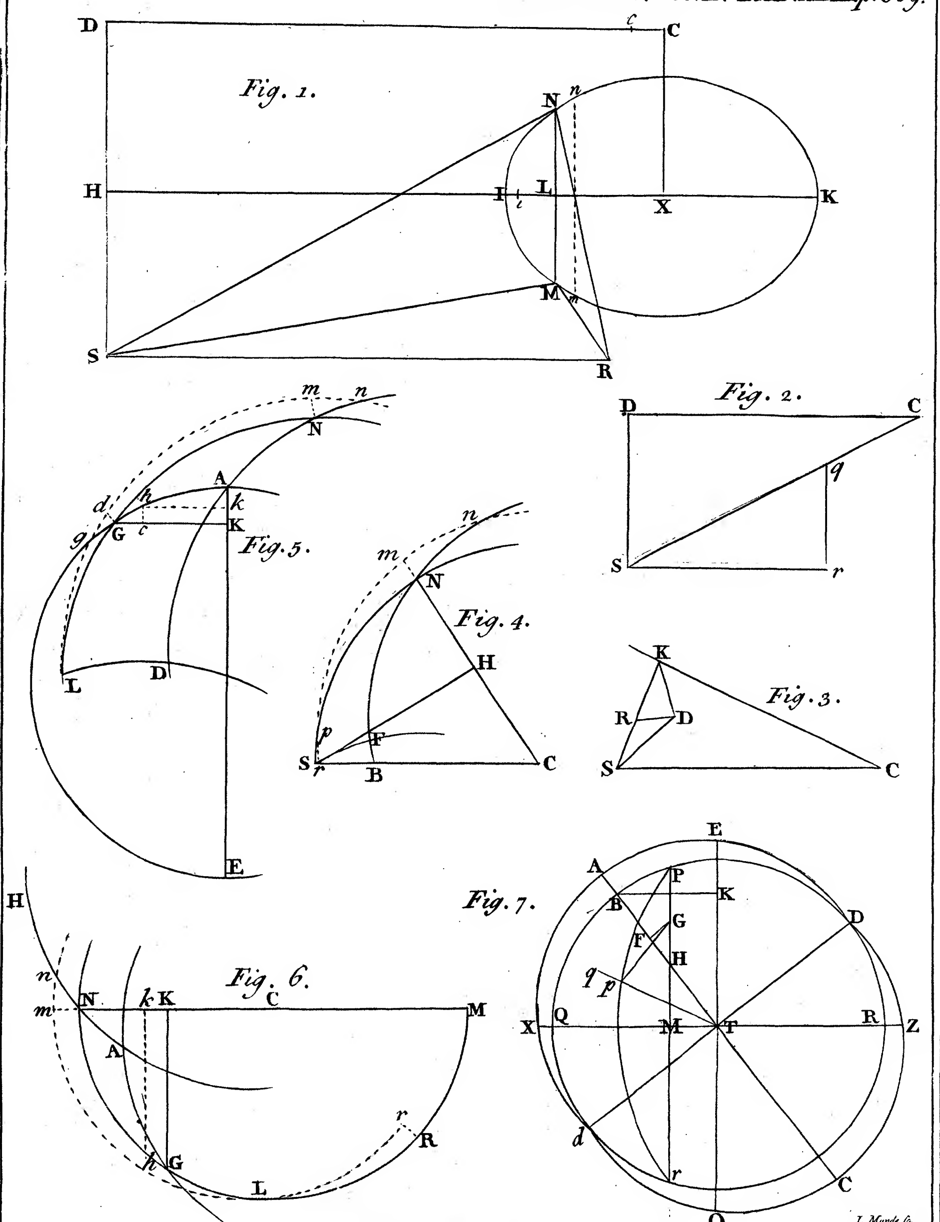
Fig. 1.
D C c H I i L X K N n M m S R
Fig. 2.
D C q S r
Fig. 3.
K R D S C
Fig. 4.
m n N H F p S r B C
Fig. 5.
m n N A d h k g G c K L D E
Fig. 6.
H n m N k K C M A r R h G L
Fig. 7.
E A P K B G D F H q p X Q M T R Z d r C O

J. Mynde sc.